Tasks 01 - Notation & Logic

Mathematics for Master Students

These problems accompany Session 01 of the tutorial. Work through them at your own pace before the next session. We start Session 02 with your questions. On the website, every problem has a fully worked solution you can expand to check yourself. The PDF version contains only the problems, so you can print it and use it as a worksheet.

The problems are staged: the first two are warm-ups, the middle ones practice the core techniques, and the last two are at the level of the mock exam.

Problem 1: Classifying Numbers

For each of the following numbers, list **all** number sets among $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$ that the number belongs to. Simplify first where possible. Remember the convention of this tutorial: $\mathbb{N} = \{1, 2, 3, ...\}$, and we write $\mathbb{N}_0$ if zero is included.

a) 7
b) -3
c) $\frac{12}{4}$
d) 0.2
e) $\sqrt{2}$
f) 0

Problem 2: Intervals, Inequalities and Absolute Value

Translate between the different ways of describing a range of numbers.

a) A supplier requires a minimum order quantity of 20 units and delivers at most 500 units per order. Write the condition on the order quantity q as a chained inequality and in interval notation.
b) Pharmaceutical goods must be stored at a temperature T (in °C) of at least 2 and strictly below 8. Write this as a chained inequality and in interval notation. Is a measured temperature of exactly 8.0 °C acceptable?
c) Write $x \in (0, 12]$ as an inequality and state in words which endpoints are included.
d) A filling machine is set to 750 ml per bottle. Quality control accepts a bottle if its content x satisfies $|x - 750| \leq 5$. Describe the set of acceptable contents as an interval.

Problem 3: Sum and Product Notation

a) Evaluate $\sum_{i=1}^{5} (2i - 1)$.
b) Evaluate $\prod_{i=1}^{4} \frac{i+1}{i}$.
c) Write $x_3 + x_4 + ... + x_{10}$ in sum notation. How many terms does the sum have?

d) A company's variable monthly costs c_t (in €) added up to $\sum_{t=1}^{12} c_t = 36,000$ last year. A new contract gives a 10% discount on all variable costs but adds a fixed service fee of €500 per month. Evaluate the new annual total $\sum_{t=1}^{12} (0.9\, c_t + 500)$, without knowing the individual c_t.

Problem 4: Warehouse Flows with Double Indices

A company ships goods from two warehouses ($i = 1, 2$) to three customers ($j = 1, 2, 3$). Let x_{ij} be the quantity (in units) shipped from warehouse i to customer j, and c_{ij} the transport cost per unit (in €) on that route:

x_{ij}	Customer 1	Customer 2	Customer 3
Warehouse 1	30	10	20
Warehouse 2	0	25	15

c_{ij}	Customer 1	Customer 2	Customer 3
Warehouse 1	2	4	3
Warehouse 2	5	1	2

a) What does x_{12} mean in words, and what is its value?
b) Write "the total quantity leaving warehouse 1" in sum notation and compute it.
c) Write "the total quantity received by customer 2" in sum notation and compute it.
d) Compute the total transport cost $\sum_{i=1}^{2} \sum_{j=1}^{3} c_{ij}\, x_{ij}$.

Problem 5: Truth Tables and De Morgan's Laws

Let p and q be two statements.

a) Complete the truth table:

p	q	$\neg p$	$p \wedge q$	$p \vee q$
T	T			
T	F			
F	T			
F	F			

b) Extend the table by two columns: $\neg(p \wedge q)$ and $\neg p \vee \neg q$. Compare them. Which law do you recognise?

c) Using that law, negate in plain English: "The shipment arrived on time **and** the invoice was correct."

d) Negate in plain English: "The container is damaged **or** it was misrouted."

Problem 6: Implications in Quality Management

A logistics company works with the rule: "If a supplier is certified, then it has passed the quality audit."

a) Define suitable propositions p and q and write the rule in logical notation.
b) State the **converse**, the **inverse** and the **contrapositive** of the rule, each in notation and in plain English. Which of the three is equivalent to the original rule?
c) For each situation, state what you can conclude from the rule, if anything:
   i) Supplier A has passed the quality audit.
   ii) Supplier B has **not** passed the quality audit.
   iii) Supplier C is **not** certified.
d) Fill in *necessary* or *sufficient*: "Passing the audit is a _____ condition for being certified. Being certified is a _____ condition for having passed the audit."

Problem 7: Negating Quantified Statements

For each statement below, first write it in quantifier notation (introduce a suitable set and predicate), then write the **negation** both formally and in plain English.

a) "Every delivery last week arrived on time."
b) "Some warehouse operates above 90% of its capacity."
c) "There exists a natural number n with $n^2 = 20$." Additionally: which is true, the statement or its negation? Justify your answer.

Problem 8: True or False, One Number Decides

A fellow student preparing for the mock exam writes down the four claims below. For each claim, decide whether it is **true** or **false** and justify your decision with a single well-chosen number (or pair of numbers). Remember from the lecture: to disprove a $\forall$ statement, **one counterexample** suffices; to prove an $\exists$ statement, **one witness** suffices.

a) $\forall x \in \mathbb{R} : x^2 \geq x$
b) $\forall a, b \in \mathbb{R} : |a + b| = |a| + |b|$
c) $\exists n \in \mathbb{N} : n^2 + n = 12$
d) "Every rational number is an integer."