Session 04 - Differentiation

Mathematics for Master Students

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Recap & Your Questions

Session 03 in a Nutshell

- A function assigns each input exactly one output
- The gallery: linear, quadratic, polynomial, rational, exponential, logarithm
- Maximal domain, red flags: division by zero, roots of negatives, logs of non-positives
- Composition chains functions (order matters); inverses undo
- Convex vs concave: where the chord lies relative to the graph
- Today: how fast does the output change, $f(a+h)$ finally pays off

Common Issues in the Homework

- **One restriction forgotten:** with $\frac{\sqrt{x+1}}{x-2}$, *both* the root ($x \geq -1$) and the denominator ($x \neq 2$) restrict: collect every red flag, then intersect
- **Composition order:** $(f \circ g)(x)$ means g acts **first**. Several solutions applied f first
- **The log-inequality flip:** dividing by $\ln 0.85 < 0$ flips the inequality. Dividing by a negative number always does

...

> Tip
>
> Which tasks gave you trouble? Bring them up **now**. We take the time to go through them before we move on to today's topic.

Warm-Up: Compose and Restrict

> Question
>
> Let $f(x) = \sqrt{x}$ and $g(x) = 2x + 7$. Compute $(f \circ g)(1)$ and find the domain of $f \circ g$.

...

- Inside out: $(f \circ g)(1) = f(g(1)) = f(9) = 3$
- The root needs $g(x) \geq 0$: $2x + 7 \geq 0$, so $x \geq -\frac{7}{2}$
- Domain of $f \circ g$: $[-\frac{7}{2}, \infty)$, the inner function must land where the outer one is defined

Today's Plan

- **Rates of change**: from secant lines to the tangent and the derivative
- **Rules of differentiation**: power, sum, product, quotient, chain, e^x and $\ln x$
- **Second derivative**: the convex/concave intuition made precise
- **Marginal analysis**: marginal cost, marginal revenue, elasticity of demand

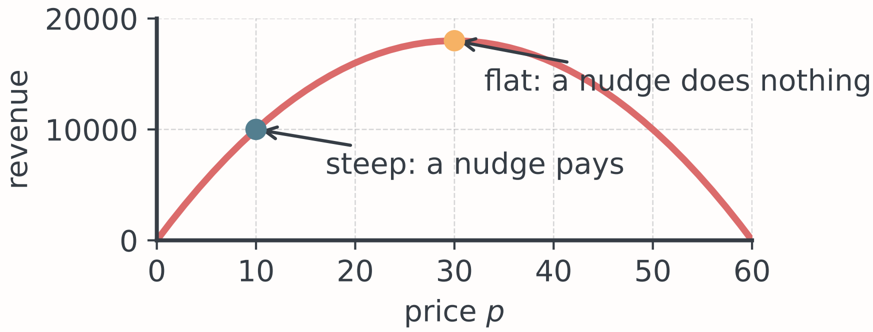
Rates of Change

How Fast Does Revenue React?

Session 3 modelled revenue as a function of price: $R(p) = 1200p - 20p^2$.

- A pricing decision is rarely "pick p from scratch", it is nudge the current price a little
- Central question: if I move the price from p to $p + h$, how fast does revenue react?
- The answer depends on **where** we stand on the curve. Let's look

The Same Nudge, Very Different Effects



. . .

Raising the price by €1 gains exactly €780 of revenue at $p = 10$, but *loses* €20 at $p = 30$. One curve, very different local behaviour.

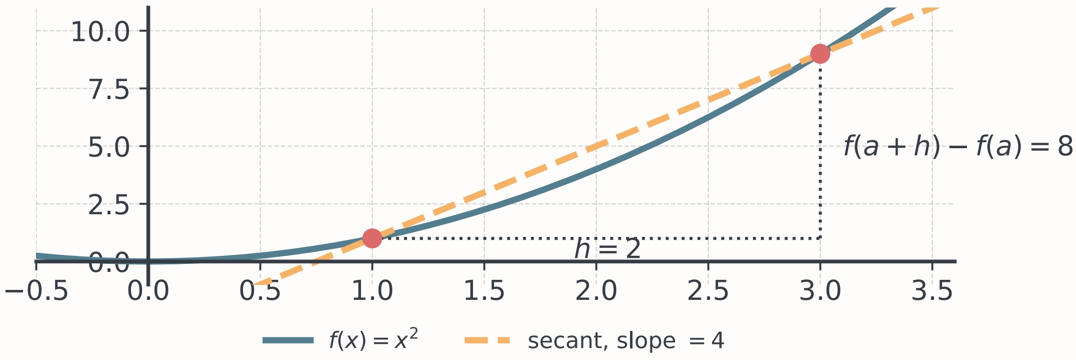
The Average Rate of Change

How much does f change per unit of input, between a and $a + h$?

$$\frac{f(a+h) - f(a)}{h} \qquad \text{the [difference quotient] .highlight}$$

- Change in **output** divided by change in **input**, "rise over run"
- The numerator is exactly Session 3's expression $f(a+h)$ minus $f(a)$, the raw material, cashed in
- For a linear function this is always the slope m. For a curved graph it **depends on a and h**

Geometry: The Secant Line



. . .

The difference quotient is the slope of the secant line through $(a, f(a))$ and $(a + h, f(a+h))$: here $\frac{9-1}{2} = 4$ for $f(x) = x^2$ from $a = 1$ to $a + h = 3$.

Your Turn: Compute the Secant Slope

i Question

For $f(x) = x^2$, what is the difference quotient from $a = 2$ to $a + h = 4$? Type the number in the chat.

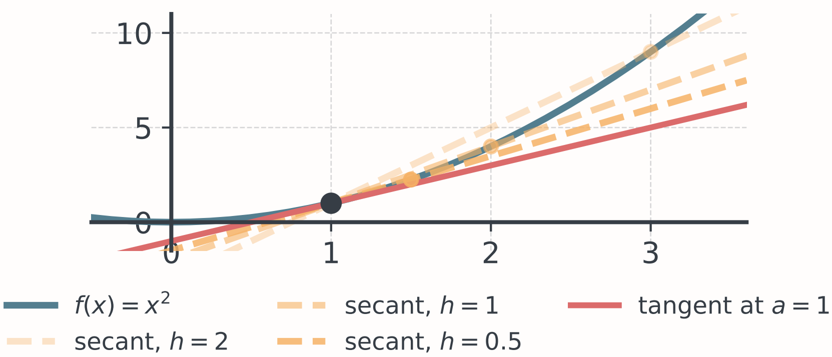
. . .

$$\frac{f(4) - f(2)}{4 - 2} = \frac{16 - 4}{2} = 6$$

. . .

The secant from 1 to 3 had slope 4, this one has slope 6: the parabola gets steeper as we move right, exactly what the next slides confirm.

From Secant to Tangent



. . .

Keep a fixed and pull the second point closer: the secants tilt toward one limiting line, the **tangent** at a. Its slope is the rate of change *right at* a.

Letting h Shrink

For $f(x) = x^2$ at $a = 1$, the secant slope is $\frac{(1+h)^2-1}{h}$:

h	1	0.5	0.1	0.01	0.001
secant slope	3	2.5	2.1	2.01	2.001

- As h shrinks toward 0, the slopes settle on the value 2
- We write $\lim_{h\to 0}$ for "the value it settles on as h shrinks toward 0"
- That is all the limit machinery we need, no formal theory required in this tutorial

The Derivative

The derivative of f at a is the instantaneous rate of change:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

- **Geometric reading:** slope of the tangent line at $(a, f(a))$
- **Business reading:** how fast the output reacts to a tiny nudge at a
- Computed at every point, it defines a new function f', the derivative of f
- Positive f': increasing; negative f': decreasing; the sign is a **trend report**

Your Turn: Read the Sign

> **i Question**
>
> You are told $f'(a) = -3$. What does that say?
>
> **(a)** f is decreasing at a, with tangent slope -3
>
> **(b)** $f(a) = -3$
>
> **(c)** f has a minimum at a
>
> **(d)** f drops by exactly 3 between a and $a+1$

...

(a): the derivative is a *slope*, and a negative slope means the graph falls at a.

...

(d) is only approximately right, and only for a nearly straight graph: the tangent slope describes the trend *at* a, not a whole unit further on. (b) confuses the value with the slope.

Worked Example: Differentiating $f(x) = x^2$

- **Expand** (Session 3 did this): $f(a+h) = (a+h)^2 = a^2 + 2ah + h^2$

- **Subtract:** $f(a+h) - f(a) = 2ah + h^2$
- **Divide by h:** $\frac{2ah+h^2}{h} = 2a + h$
- **Shrink h toward 0:** $f'(a) = 2a$

...

> Tip
>
> Check against the table: at $a = 1$ we get $f'(1) = 2$, exactly where the secant slopes settled. And at $a = 3$: slope 6, the parabola gets steeper as we move right.

Notation for Derivatives

All of these mean the same thing:

Notation	Read as
$f'(x)$	"f prime of x"
$\frac{df}{dx}$	"derivative of f with respect to x"
$\frac{d}{dx}f(x)$	"$\frac{d}{dx}$ applied to $f(x)$"

...

> Tip
>
> $\frac{df}{dx}$ deliberately looks like a difference quotient: it reminds you where the derivative comes from. We mostly write $f'(x)$; with several variables around (e.g. $C(q)$), the $\frac{dC}{dq}$ form says *with respect to what*.

The Tangent as a Local Approximation

Near a, the graph and its tangent are almost indistinguishable:

$$f(a+h) \approx f(a) + f'(a) \cdot h \qquad \text{for small} \quad h$$

- The difference quotient rearranged: $f'(a) \approx \frac{f(a+h)-f(a)}{h}$, multiply by h and move $f(a)$ across
- **Example:** $f(x) = x^2$ at $a = 3$, $f'(3) = 6$: $f(3.1) \approx 9 + 6 \cdot 0.1 = 9.6$, exact value 9.61
- Business reading: "one more unit changes the output by about $f'(a)$", the idea behind marginal cost, later today

Your Turn: Average vs Instantaneous

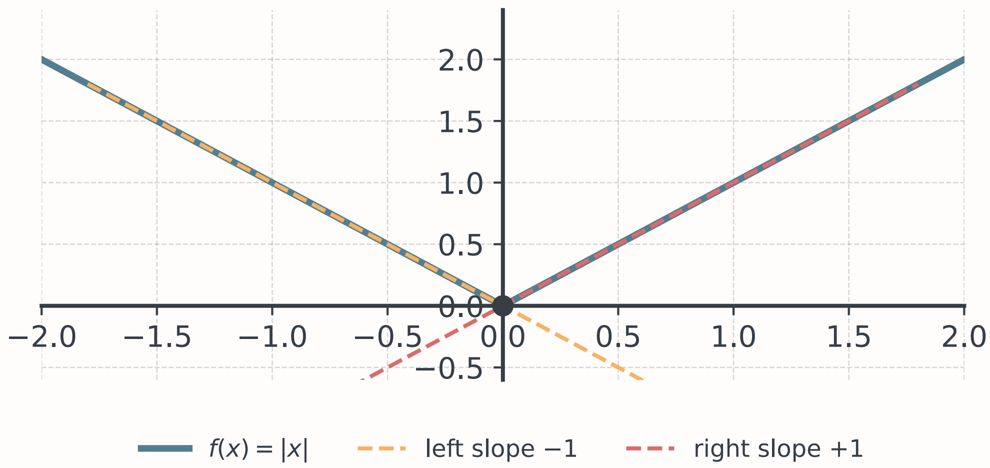
i Question

A courier van covers 150 km in 2 hours. Halfway through, a speed camera clocks it at 90 km/h. Which number is an *average* rate of change, which is a *derivative*, and can both be correct at once?

. . .

- $\frac{150}{2} = 75$ km/h is a difference quotient, distance change over time change
- The camera reading is the instantaneous rate, the derivative of distance at that moment
- Both are correct: the average smooths out faster and slower stretches

Not Every Function Has a Derivative



Kinks, Jumps, and Smoothness

- At the kink, secants from the left settle on -1, from the right on $+1$: no single tangent, no derivative at 0
- **Jumps** are even worse: no tangent can be drawn across a gap
- Intuition: differentiable = smooth graph: no kinks, no jumps (tiered shipping tariffs kink at the breakpoints)

Rules of Differentiation

Constants and the Power Rule

Nobody computes difference quotients all day, rules do the work:

- **Constants:** $f(x) = c$ has $f'(x) = 0$, a horizontal line has slope zero
- **Power rule:** $f(x) = x^n$ has $f'(x) = nx^{n-1}$
- Recipe: exponent down front, then reduce the exponent by one

- **Example:** $(x^5)' = 5x^4$, and our $(x^2)' = 2x$ is the case $n = 2$

The Power Rule Takes All Exponents

The rule also covers negative and fractional exponents, rewrite first:

- $\frac{1}{x} = x^{-1}$, so $\left(\frac{1}{x}\right)' = -1 \cdot x^{-2} = -\frac{1}{x^2}$
- $\sqrt{x} = x^{1/2}$, so $\left(\sqrt{x}\right)' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

...

> Tip
>
> Session 3's exponent rules pay off: **rewrite as a power**, differentiate, translate back. Fractions and roots stop being special cases.

Your Turn: Apply the Power Rule

> Question
>
> Let $f(x) = x^4$. What is $f'(2)$? Type the number. Bonus: differentiate $g(x) = \frac{1}{x^2}$.

...

$$f'(x) = 4x^3, \qquad f'(2) = 4 \cdot 8 = 32$$

...

$$g(x) = x^{-2}, \qquad g'(x) = -2x^{-3} = -\frac{2}{x^3}$$

...

> Common Mistake
>
> $f'(2) = 4 \cdot 2^4 = 64$ forgets to reduce the exponent. Exponent down front **and** one less on top.

Sums and Constant Multiples

$$(c \cdot f)'(x) = c \cdot f'(x) \qquad (f + g)'(x) = f'(x) + g'(x)$$

- Together: differentiate term by term, constants tag along
- **Example**, a cost function: $C(q) = 0.5q^2 + 10q + 200$
- $C'(q) = 0.5 \cdot 2q + 10 + 0 = q + 10$
- The fixed costs 200 vanish: they do not react to producing one unit more

...

Every polynomial is now differentiable in one line. Keep $C'(q) = q + 10$ in mind, it returns later today.

A Cubic Revenue Function

Revenue from a promotion budget x: $R(x) = -0.5x^3 + 6x^2 + 20x$

- Term by term: $(-0.5x^3)' = -1.5x^2$, $(6x^2)' = 12x$, $(20x)' = 20$
- So $R'(x) = -1.5x^2 + 12x + 20$
- At $x = 2$: $R'(2) = -6 + 24 + 20 = 38 > 0$, revenue still rises with the budget
- At $x = 10$: $R'(10) = -150 + 120 + 20 = -10 < 0$, the budget has overshot

. . .

The sign of R' is the trend report: positive means “spend more”, negative means “you overshot”.

Your Turn: Differentiate Term by Term

ℹ Question

A cost function is $C(q) = 0.1q^2 + 4q + 50$. What is $C'(20)$? Type the number.

. . .

$$C'(q) = 0.2q + 4, \qquad C'(20) = 4 + 4 = 8$$

. . .

At 20 units, cost grows at €8 per unit, while the fixed cost 50 has disappeared from C' entirely: it never reacts to output.

The Product Rule

$$(f \cdot g)'(x) = f'(x)\, g(x) + f(x)\, g'(x)$$

- “Derivative of the first times the second, plus the first times derivative of the second”
- **Example:** $u(x) = (x^2 + 1)(3x - 2)$
- $u'(x) = 2x \cdot (3x - 2) + (x^2 + 1) \cdot 3 = 9x^2 - 4x + 3$
- Check: expanding first gives $u(x) = 3x^3 - 2x^2 + 3x - 2$, so $u'(x) = 9x^2 - 4x + 3$ ✓

. . .

⚠ Common Mistake

$(f \cdot g)' \neq f' \cdot g'$, try $f(x) = g(x) = x$: the product x^2 has derivative $2x$, not 1.

The Quotient Rule

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$$

- Like the product rule, but with a minus, the numerator's order matters
- **Example**, Session 3's average cost per unit: $A(q) = \frac{200+5q}{q}$
- $A'(q) = \frac{5 \cdot q - (200+5q) \cdot 1}{q^2} = -\frac{200}{q^2}$
- Always negative: average cost falls as volume grows: economies of scale, now proven

The Chain Rule

For a composition $(f \circ g)(x) = f(g(x))$, Session 3's chaining:

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

- Recipe: outer derivative (inner left untouched) **times** inner derivative
- **Example:** $v(x) = (2x^2+1)^3$, outer $(\cdot)^3$, inner $2x^2+1$
- $v'(x) = 3(2x^2+1)^2 \cdot 4x = 12x\,(2x^2+1)^2$

. . .

⚠ Common Mistake

Stopping at $3(2x^2+1)^2$, the inner derivative $4x$ must come along. Forgetting it is *the* classic chain-rule slip.

Your Turn: Spot the Missing Factor

i Question

What is the derivative of $(3x+2)^5$?

(a) $5(3x+2)^4$ **(b)** $15(3x+2)^4$

(c) $5 \cdot 3^4$ **(d)** $15x\,(3x+2)^4$

. . .

(b): outer derivative $5(3x+2)^4$, times the inner derivative 3.

. . .

(a) forgot the inner factor, (c) differentiated the bracket away, (d) invented an x: the inner derivative of $3x+2$ is the constant 3, nothing more.

Chain Rule in Business: Rates Multiply

A warehouse's stock grows over time, and storage cost depends on stock:

$$q(t) = 30 + 4t \ \text{ pallets after } \ t \ \text{ weeks,} \qquad S(q) = 0.2q^2 \ \text{ cost in €}$$

. . .

How fast does the **cost grow over time**? Differentiate the chain $S(q(t))$:

$$\frac{dS}{dt} = S'(q) \cdot q'(t) = 0.4q \cdot 4 = 1.6\,(30 + 4t)$$

...

- At $t = 5$: stock $q = 50$, and cost rises at $1.6 \cdot 50 = 80$, €80 per week
- The chain rule multiplies the rates along the chain: € per pallet times pallets per week

Your Turn: Pick the Rule

i Question

Which rule differentiates each function, and what are the derivatives?

$$\text{(a)}\ (5x+1)^4 \qquad \text{(b)}\ \ x^3(2x-7) \qquad \text{(c)}\ \ \frac{x}{x^2+1}$$

...

- **(a) Chain rule**, outer $(\cdot)^4$, inner $5x + 1$: derivative $4(5x + 1)^3 \cdot 5 = 20(5x + 1)^3$
- **(b) Product rule**, or expand to $2x^4 - 7x^3$ and use the power rule: both give $8x^3 - 21x^2$
- **(c) Quotient rule**: $\left(\frac{x}{x^2+1}\right)' = \frac{1 \cdot (x^2+1) - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$

Derivatives of e^x and $\ln x$

- $(e^x)' = e^x$, the exponential is its own derivative: at every point, slope = height
- $(\ln x)' = \frac{1}{x}$, steep near 0, ever flatter afterwards: diminishing returns
- Other bases pick up a factor: $(a^x)' = a^x \ln a$
- e is exactly the base where that factor is $\ln e = 1$, the tidy one
- With the chain rule: $(e^{3x})' = e^{3x} \cdot 3 = 3e^{3x}$

Worked Example: Mixing the Rules

Differentiate $u(x) = x \cdot e^x$ and $v(x) = \ln(x^2 + 1)$.

- u is a **product**: $u'(x) = 1 \cdot e^x + x \cdot e^x = (1 + x)\, e^x$
- v is a **chain**: outer $\ln(\cdot)$ with derivative $\frac{1}{(\cdot)}$, inner $x^2 + 1$ with derivative $2x$
- $v'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

...

Tip

Name the structure first, product or chain, and the rule follows. Two rules can also nest: $(xe^{2x})'$ is a product whose second factor needs the chain rule.

Your Turn: Exponential and Log

> **i Question**
>
> Differentiate $f(x) = e^{2x}$ and $g(x) = \ln(5x)$. Type both derivatives in the chat.

...

- $f'(x) = e^{2x} \cdot 2 = 2e^{2x}$: outer $e^{(\cdot)}$, inner derivative 2
- $g'(x) = \frac{1}{5x} \cdot 5 = \frac{1}{x}$: the 5 cancels
- No surprise: $\ln(5x) = \ln 5 + \ln x$, and the constant $\ln 5$ differentiates to zero

The Superpower of e

Session 3's continuous growth: $K(t) = K_0 \cdot e^{rt}$. Differentiate (chain rule):

$$K'(t) = K_0\, e^{rt} \cdot r = r \cdot K(t)$$

- The growth rate is proportional to the current level: at every instant, capital grows at r times itself
- That is the defining property of exponential growth: interest earns interest
- This is *why* e rules finance and growth models, the promised superpower

The Rule Table

Building blocks

$f(x)$	$f'(x)$
c	0
x^n	nx^{n-1}
e^x	e^x
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$

Combination rules

Rule	Formula
Constant multiple	$(cf)' = cf'$
Sum	$(f+g)' = f' + g'$
Product	$(fg)' = f'g + fg'$
Quotient	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
Chain	$(f(g))' = f'(g) \cdot g'$

...

> Tip
>
> These ten lines differentiate **every function in Session 3's gallery**: practice comes with the tasks, the table lives on the cheatsheet.

Higher-Order Derivatives & Curvature

Differentiating Twice

f' is again a function, so differentiate again:

$$f''(x) = (f')'(x) \qquad \text{also written } \frac{d^2 f}{dx^2}$$

- f' reports how fast f changes; f'' reports how fast the slope changes
- **Example:** $f(x) = x^3 - 3x$, Session 3's cubic
- First: $f'(x) = 3x^2 - 3$; then: $f''(x) = 6x$
- Third, fourth, … derivatives exist too: for us, two is all we need

Your Turn: Differentiate Twice

> Question
>
> Let $f(x) = 2x^3 - 5x$. What is $f''(3)$? Type the number.

...

$$f'(x) = 6x^2 - 5, \qquad f''(x) = 12x, \qquad f''(3) = 36$$

...

Positive: at $x = 3$ the slope is increasing, the graph bends upward, convex there. The next slide makes that reading official.

Curvature: The Second-Derivative Test

Session 3's chord intuition, now with a formal test on an interval:

- $f''(x) \geq 0$ everywhere $\Rightarrow$ convex: slope increasing, graph bends **upward**, chords above
- $f''(x) \leq 0$ everywhere $\Rightarrow$ concave: slope decreasing, graph bends **downward**, chords below
- $f(x) = x^2$: $f''(x) = 2 > 0$: convex everywhere, as the chords showed
- $f(x) = \ln x$: $f''(x) = -\frac{1}{x^2} < 0$, concave everywhere: diminishing returns, certified

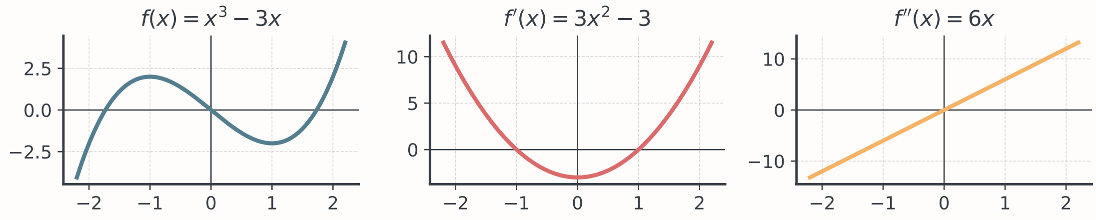
Curvature in Business: Diminishing Returns

Sales from an advertising budget a (in €1000): $S(a) = 100\sqrt{a}$

- $S'(a) = \frac{50}{\sqrt{a}} > 0$: more budget, more sales, always

- $S''(a) = -\frac{25}{a^{3/2}} < 0$: but each extra €1000 buys less than the previous one
- Concave everywhere: the classic shape of **diminishing returns**
- The first euro of advertising works hardest, the millionth barely moves the needle

Reading f, f' and f'' Together



. . .

- For $x < 0$: $f'' < 0$: slope falling, f bends downward (concave)
- For $x > 0$: $f'' > 0$: slope rising, f bends upward (convex)
- At $x = 0$ the bend switches, an inflection point; Session 5 turns these signs into an optimisation tool

Your Turn: What Does $C''(q) > 0$ Mean Here?

ℹ Question

A warehouse's handling cost $C(q)$ satisfies $C'(q) > 0$ *and* $C''(q) > 0$ for all realistic q. What does each sign say about the next pallet?

. . .

- $C'(q) > 0$: handling more pallets costs more, the cost curve rises
- $C''(q) > 0$: each *additional* pallet costs more than the previous one, the rise steepens
- Business reading: the warehouse strains against capacity, the cost curve is **convex**

Marginal Analysis

Marginal Cost

Economists call $C'(q)$ the marginal cost at output level q:

- Exact meaning: the **instantaneous rate** at which cost grows at q
- Practical meaning: approximately the cost of one more unit
- Why: with $h = 1$ in the difference quotient, $C'(q) \approx \frac{C(q+1)-C(q)}{1}$
- One unit is a *small* nudge next to hundreds produced, so the tangent slope is an excellent stand-in

Worked Reading: What Does $C'(50)$ Say?

Take today's cost function $C(q) = 0.5q^2 + 10q + 200$, so $C'(q) = q + 10$.

- $C'(50) = 60$, at 50 units, cost grows at €60 per unit

- Reading: the **51st unit** costs approximately €60 to produce
- Exact check: $C(51) - C(50) = 2010.5 - 1950 = 60.5$, the approximation is off by 50 cents
- And $C''(q) = 1 > 0$: marginal cost is increasing, every further unit is dearer

. . .

> Tip
>
> Mind the units: $C'(q)$ is **€ per unit**, not €: a rate, not a cost.

Your Turn: The 101st Unit

> Question
>
> Same cost function, $C(q) = 0.5q^2 + 10q + 200$. Approximately what does the **101st** unit cost? Type the number.

. . .

$$C'(100) = 100 + 10 = 110 \qquad \text{so about €110}$$

. . .

Exact check: $C(101) - C(100) = 6310.5 - 6200 = 110.5$. Again the tangent slope misses by 50 cents, and it is far quicker than the subtraction.

Marginal Revenue

The same idea on the revenue side, sell q units, earn $R(q)$:

- Marginal revenue $R'(q)$: approximately the revenue from **one more unit** sold
- It usually *falls* with q, selling more requires lowering the price
- Producing one more unit pays off while $R'(q) > C'(q)$, the extra revenue beats the extra cost
- Where the two rates meet lies the profit-maximal output, Session 5 makes this precise

Elasticity of Demand

Rates in **units** depend on the units, pricing needs percentages. For a demand function $D(p)$:

$$\varepsilon = \frac{p}{D(p)} \cdot D'(p)$$

- Reading: a **1% price increase** changes demand by about ε percent
- Demand falls in price, so ε is negative
- $|\varepsilon| > 1$: elastic, demand overreacts to price
- $|\varepsilon| < 1$: inelastic, demand barely reacts

Worked Example: Pricing a Delivery Service

Weekly demand for a same-day delivery service: $D(p) = 500 - 10p$, so $D'(p) = -10$.

- **At $p = 20$:** $D(20) = 300$, so $\varepsilon = \frac{20}{300} \cdot (-10) \approx -0.67$
- Inelastic: raising the price 1% loses only 0.67% of demand, revenue rises
- **At $p = 30$:** $D(30) = 200$, so $\varepsilon = -1.5$, elastic: a price increase now backfires
- The switch sits at $\varepsilon = -1$ (here $p = 25$), exactly where revenue $p \cdot D(p)$ peaks

Your Turn: Raise the Price or Not?

> **i Question**
>
> At the current price, a freight forwarder estimates the elasticity of demand at $\varepsilon = -0.4$. Elastic or inelastic, and what does a small price increase do to revenue?

. . .

- $|\varepsilon| = 0.4 < 1$: inelastic, customers barely react
- A 1% price increase loses only about 0.4% of demand
- Price up, demand almost unchanged: revenue increases, typical for services customers depend on

Closing

Key Takeaways

- The derivative $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ is the instantaneous rate of change, the tangent's slope
- Kinks and jumps break differentiability, smooth graphs are safe
- Power, sum, product, quotient and chain rule handle every gallery function
- e^x is its own derivative, growth proportional to the current level
- f'' measures curvature: $f'' \geq 0$ convex, $f'' \leq 0$ concave, the chord intuition, formalised
- $C'(q) \approx$ cost of one more unit; elasticity ε compares percentage reactions

Skip order if running long: 1. The Superpower of e (fold its punchline into the $e^x/\ln x$ slide), 2. Marginal Revenue (one sentence on the worked-reading slide), 3. Not Every Function Has a Derivative (keep only the kink bullet), 4. compress The Same Nudge plot to a verbal example.

That's it for today.

Any Questions?

Until the Next Session

- Work through the Tasks: the drilling happens there, with worked solutions

- Check yourself with the Self-Test quiz
- Keep the Cheatsheet next to you, the rule table lives on it
- Note down anything unclear, we start next session with **your questions**

. . .

> Tip
>
> When differentiating, name the structure first: sum, product, quotient or chain? The right rule follows automatically.

Preview: Session 05: Single Variable Optimization

- Central question: where is a function largest or smallest?
- The recipe: set f' to **zero** (flat tangent) and ask f'' whether it is a peak or a valley
- Best output, best price, best order quantity
- Today's rules become the toolkit, differentiation was the warm-up

. . .

See you there, and bring your questions!

Literature & Further Reading

- These sessions cover the essentials: textbooks offer more depth and practice
- Start with K. Sydsæter, P. J. Hammond, and A. Strøm [1] or I. Jacques [2]; full recommendations on the tutorial's literature page

Bibliography

[1] K. Sydsæter, P. J. Hammond, and A. Strøm, *Essential Mathematics for Economic Analysis*, 4th ed. Harlow: Pearson, 2012.

[2] I. Jacques, *Mathematics for Economics and Business*, 8th ed. in Always Learning. Harlow: Pearson, 2015, p. 660.