Session 01 - Notation & Logic

Mathematics for Master Students

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Introduction

About Me

- **Field:** Optimising and simulating complex systems
- **Languages of choice:** Julia, Python and Rust
- **Interest:** Modelling, simulations, machine learning
- **Teaching:** OR, algorithms, programming & mathematics
- **Contact:** vlcek@beyondsimulations.com

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> Tip
>
> I really appreciate active participation and questions: the more you ask, the more useful these sessions become for you!

Our Plan for Today

- How this tutorial works and why it exists
- How to make good use of AI while studying

- **Notation**: the vocabulary of mathematics
- **Logic**: the grammar of mathematical reasoning
- Your homework until the next session

Tutorial Structure

Session	Topic	Date	Time	Room
1	Notation & Logic	Tue 8 Sep 2026	18:00–19:30	Auditorium
2	Sets	Tue 15 Sep 2026	18:00–19:30	Auditorium
3	Functions	Tue 22 Sep 2026	18:00–19:30	Zoom
4	Differentiation	Tue 29 Sep 2026	18:00–19:30	Zoom
5	Single Variable Optimization	Tue 6 Oct 2026	18:00–19:30	Auditorium
6	Systems of Linear Equations	Tue 13 Oct 2026	18:00–19:30	Auditorium
7	Mock Exam *(voluntary!)*	Tue 27 Oct 2026	14:30–16:00	Auditorium

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Six sessions of 90 minutes plus a voluntary mock exam: a compact refresher of the mathematics used in your M.Sc. courses.

Why These Topics?

- Your KLU courses **assume** you can read and use mathematics
- Lecturers will not pause to explain notation or reasoning
- The topics of this tutorial were chosen by checking where they appear in your first- and second-semester courses
- The next two slides show the result of that mapping

Maths Topics in Your KLU Courses I

Semester 1

Course	Notation	Logic	Sets	Functions	Differentiation	Linear Systems	Optimization
Statistics & Econometrics	✓	✓	✓			✓	
Business Logistics & SCM	✓			✓	✓		✓
Fundamentals of Business Analytics	✓			✓	✓	✓	✓
International Economics	✓			✓	✓	✓	✓
Strategic Issues in SCM	✓	✓				✓	
Fundamentals of Data Science	✓	✓	✓	✓	✓		

Maths Topics in Your KLU Courses II

Semester 2

Course	Notation	Logic	Sets	Functions	Differentiation	Linear Systems	Optimization
Transportation & Distribution	✓	✓	✓	✓	✓	✓	✓
Global Trends in Human Resources		✓				✓	
Economics of Digital Transformation				✓	✓		
Economics of Business Strategy	✓			✓	✓	✓	✓
Business Strategy & Sustainability				✓			

. . .

Today’s topics, Notation and Logic, appear in almost every course.

How This Tutorial Works

Each session comes with four materials:

- **Lecture**: what we discuss together in class
- **Tasks**: homework problems with worked solutions
- **Self-Test**: a short online quiz to check yourself
- **Cheatsheet**: a compact reference to keep at hand

. . .

You work on the tasks **between sessions**. Every session starts with a recap and your questions.

No Grades, Just Preparation

- This tutorial is not graded: it is purely preparatory
- Nobody tracks your answers or your progress
- The only goal: you follow your M.Sc. courses **with ease**
- The mock exam in Session 7 is **voluntary**, and we work through it together

. . .

> Tip
>
> Ask any question you have, no matter how *simple* it may seem. If you are lost, even after something was explained multiple times, ask again: that is exactly what this tutorial is for!

Tutorial Materials

- All materials are hosted online at math-master.tobiasvlcek.com
- Lectures are available as **web page**, **slides**, and **PDF**
- Tasks can be printed as a worksheet: solutions are on the website
- If you find good additional resources, share them with the group!

Artificial Intelligence

How to Use AI in This Tutorial

- Feel free to use AI tools to **support your understanding**
- A tutorial chatbot is available **directly on the tutorial website**
- It is designed to guide your thinking, not to hand you solutions
- There is no graded exam here, so the rule is simple:

. . .

> Tip
>
> Use AI to **learn**, not to outsource your thinking. If the chatbot solves your homework, you have practised *prompting*, not *mathematics*.

The Tutorial Chatbot

- Click the chat bubble on any page of the tutorial website
- Or open the dedicated Math Helper page
- Ask your question **as specifically as possible**: this gives the model enough context to help you
- It is set up by me for this tutorial and will nudge you with **hints** instead of full solutions
- Please don't share personal information in the chat

LLMs and Mathematics

- Large language models (LLMs) are pattern-based
- They *predict text*, they do not *compute*
- They are **great at explaining concepts** and rephrasing ideas
- They can confidently produce wrong calculations

. . .

> ⚠ Always question the maths output of LLMs!
>
> For actual computation, tools like Wolfram Alpha are more reliable, as they use **symbolic computation** rather than pattern matching.

Notation

Why Mathematical Notation?

Mathematics is a language, with vocabulary and grammar:

- It expresses complex ideas **precisely** and **compactly**
- It is understood **worldwide**, independent of your mother tongue
- A constraint like $\sum_{t=1}^{12} c_t \leq B$ says in one line what takes a paragraph in English
- Reading it fluently is a **skill**, and skills can be trained

The Number Sets

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

- $\mathbb{N} = \{1, 2, 3, \ldots\}$: natural numbers (counting: units, trucks)
- $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$: integers (profits *and* losses)
- $\mathbb{Q} = \left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$: rational numbers (fractions)
- $\mathbb{R}$, real numbers: all points on the number line, including $\sqrt{2}$ and π

. . .

> ℹ Note
>
> Some books include 0 in $\mathbb{N}$. **In this tutorial,** $\mathbb{N} = \{1, 2, 3, \ldots\}$, and we write $\mathbb{N}_0$ if zero is included.

Your Turn: Rational or Not?

> ℹ Question
>
> Which of these numbers are **rational**: $\quad 0.75, \quad \sqrt{2}, \quad \frac{7}{3}, \quad \pi$?

. . .

- $0.75 = \frac{3}{4}$ and $\frac{7}{3}$ are rational: both can be written as a fraction of integers
- $\sqrt{2}$ and π are **irrational**: they are real, but no fraction equals them exactly

...

> ⚠ Common Mistake
>
> $\pi \approx \frac{22}{7}$, but $\pi \neq \frac{22}{7}$: an approximation is not an equality!

Interval Notation

Intervals describe connected ranges of real numbers:

- **Closed:** $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$, endpoints included
- **Open:** $(a, b) = \{x \in \mathbb{R} : a < x < b\}$, endpoints excluded
- **Half-open:** $[a, b)$ includes a but excludes b
- **Unbounded:** $[0, \infty)$, all non-negative reals

...

> Tip
>
> **Square bracket** $[$ means the endpoint is **included**, **round parenthesis** $($ means it is **excluded**. Think of the bracket as "grabbing" the endpoint. ∞ always gets a parenthesis: it is not a number you could include.

Your Turn: Write the Interval

> **i** Question
>
> A warehouse accepts pallets weighing **at least** 200 kg and **strictly less than** 1000 kg. Write the admissible weight w as an interval.

...

$$w \in [200, 1000)$$

...

> ⚠ Common Mistake
>
> "At least" includes the endpoint (square bracket), "strictly less than" excludes it (parenthesis). $(200, 1000]$ has the brackets on the wrong sides: it would reject a 200 kg pallet and accept a 1000 kg one.

Inequality Notation

Symbol	Meaning	Business example
$<$	strictly less than	$x < 100$ (below the threshold)
$\leq$	less than or equal	$x \leq 500$ (capacity limit)
$\geq$	greater than or equal	$q \geq 20$ (minimum order quantity)
$\neq$	not equal to	$\text{supply} \neq \text{demand}$

...

Inequalities can be chained: $0 \leq x \leq 500$

means "x is at least 0 **and** at most 500", as an interval: $x \in [0, 500]$.

Absolute Value

The absolute value $|x|$ is the **distance** of x from zero:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

- $|5| = 5$ and $|-5| = 5$: distance is never negative
- $|a - b|$ is the distance **between** a and b
- Business use: the forecast error $|\text{demand} - \text{forecast}|$, we care about the *size* of the miss, not its direction

Sum Notation

The symbol $\sum$ (capital sigma) compresses long sums:

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \ldots + x_n$$

- i is the index: it runs from the **lower limit** 1 to the **upper limit** n
- x_i is the summand: the expression added up for each i
- The index name is arbitrary: $\sum_{i=1}^{n} x_i = \sum_{k=1}^{n} x_k$

Sums in Business

Let c_t be your company's cost in month t. The **total annual cost** is

$$\sum_{t=1}^{12} c_t = c_1 + c_2 + \ldots + c_{12}$$

...

Two rules you will use constantly:

- **Constant factors** move out: $\sum_{t=1}^{12} 0.9 \cdot c_t = 0.9 \sum_{t=1}^{12} c_t$
- **Sums split** over addition: $\sum_{t=1}^{12} (c_t + f) = \sum_{t=1}^{12} c_t + \sum_{t=1}^{12} f = \sum_{t=1}^{12} c_t + 12 \cdot f$

...

Tip

Read $\sum$ as a small **loop**: “for each t from 1 to 12, add up ...”.

Your Turn: Evaluate a Sum

Question

Compute $\sum_{i=1}^{4} 2i$. What is the result?

...

Write out the loop, one term per index value:

$$\sum_{i=1}^{4} 2i = 2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + 2 \cdot 4 = 2 + 4 + 6 + 8 = 20$$

...

Or move the constant out first: $2\sum_{i=1}^{4} i = 2 \cdot 10 = 20$.

Product Notation

The symbol $\prod$ (capital pi) works like $\sum$, but with multiplication:

$$\prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot \ldots \cdot x_n$$

...

Business example: capital grows for three years at rates r_1, r_2, r_3:

$$K_3 = K_0 \cdot \prod_{t=1}^{3}(1 + r_t) = K_0(1 + r_1)(1 + r_2)(1 + r_3)$$

...

Growth **compounds**: that is why it is a product, not a sum.

Your Turn: Compounded Growth

Question

Revenue grows by $10\,\%$, then $20\,\%$, then $5\,\%$ over three years. By what **total** percentage has it grown?

...

$$\prod_{t=1}^{3}(1+r_t) = 1.10 \cdot 1.20 \cdot 1.05 = 1.386$$

so the total growth is $38.6\,\%$.

...

> ⚠ Common Mistake
>
> $10 + 20 + 5 = 35\,\%$ is wrong: each year's growth applies to the *already grown* revenue. Rates multiply as factors, they do not add.

Double Indices

Many logistics quantities depend on two things at once:

- x_{ij} = quantity shipped **from** warehouse i **to** customer j
- c_{ij} = transport cost per unit from i to j
- So x_{23} is the flow from warehouse 2 to customer 3
- Convention: the first index is the origin (row), the second the destination (column)

...

> Tip
>
> Whenever you meet a variable, first ask: what do its indices mean? A well-defined model always states this explicitly.

Double Sums

With n warehouses and m customers:

- Everything leaving warehouse i: $\sum_{j=1}^{m} x_{ij}$
- Everything reaching customer j: $\sum_{i=1}^{n} x_{ij}$
- **Total transport cost** over all routes:

...

$$\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}\,x_{ij}$$

...

This is the objective function of the classic **transportation problem**. You will meet it again in *Transportation & Distribution*.

Your Turn: Total Transport Cost

Question

Two warehouses, two customers. Compute $\sum_{i=1}^{2} \sum_{j=1}^{2} c_{ij}\, x_{ij}$.

	c_{i1}	c_{i2}	x_{i1}	x_{i2}
$i=1$	2	5	10	0
$i=2$	4	3	5	20

. . .

$$2 \cdot 10 + 5 \cdot 0 + 4 \cdot 5 + 3 \cdot 20 = 20 + 0 + 20 + 60 = 100$$

. . .

Tip

Bonus: everything leaving warehouse 2 is $\sum_{j=1}^{2} x_{2j} = 5 + 20 = 25$. Same data, different index fixed.

Greek Letters

Greek letters are everywhere in your courses, a quick reference:

Letter	Name	Typical use
α	alpha	significance level
β	beta	regression coefficient
δ, Δ	delta	(small) change, difference
ε	epsilon	error term, small number
λ	lambda	arrival rate

Letter	Name	Typical use
μ	mu	mean
π	pi	$3.14159...$, profit
ρ	rho	correlation
σ, Σ	sigma	std. deviation, sum
θ	theta	generic parameter

Logic

Statements and Truth Values

A statement (proposition) is a declarative sentence that is either **true** or **false**, never both.

. . .

- “Hamburg is a port city.”, a **true** statement
- “$2 + 2 = 5$”, a **false** statement
- “Last week, every shipment arrived on time.”, a statement; whether it is true depends on the facts

. . .

Tip

Logic does not care what a statement is *about*, only that it has exactly one truth value.

What Is *Not* a Statement?

- “When can we take a break?”, a **question** has no truth value
- “Load the truck!”, a **command** has no truth value
- “$x + 2 = 2x$”, an open sentence: true for $x = 2$, false otherwise

. . .

An open sentence **becomes** a statement once every variable is fixed or quantified: that is why defining your variables matters so much.

Negation

The negation $\neg p$ (“not p”) flips the truth value of p:

- p: “The warehouse is full.” $\neg p$: “The warehouse is **not** full.”
- If p is true, $\neg p$ is false, and vice versa
- Double negation cancels: $\neg(\neg p) = p$

. . .

Common Mistake

The negation of “the warehouse is full” is **not** “the warehouse is empty”! Between *full* and *empty* there is a lot of room: the negation only claims “not full”.

Conjunction and Disjunction

$p \wedge q$ (“p **and** q”) $p \vee q$ (“p **or** q”)

p	q	$p \land q$	$p \lor q$
T	T	**T**	**T**
T	F	F	**T**
F	T	F	**T**
F	F	F	F

...

> **i Note**
>
> The mathematical "or" is inclusive: $p \lor q$ is also true when *both* hold. "Dessert or coffee?" in maths means: taking both is fine, too.

De Morgan's Laws

Negation flips "and" into "or", and vice versa:

$$\neg(p \land q) \Leftrightarrow \neg p \lor \neg q \qquad \neg(p \lor q) \Leftrightarrow \neg p \land \neg q$$

...

Business example: "The shipment is on time **and** complete."

Its negation: "The shipment is late **or** incomplete": one failure is enough.

...

> **Tip**
>
> You will meet the same pattern again in a moment: negating a statement swaps $\forall$ and $\exists$, just like it swaps $\land$ and $\lor$.

Implication

$p \Rightarrow q$: "**if** p, **then** q", p is the premise, q the conclusion.

p	q	$p \Rightarrow q$
T	T	**T**
T	F	F
F	T	**T**
F	F	**T**

...

Think of a promise

"If your order exceeds €500 (p), shipping is free (q)." The promise is only **broken** when the order exceeds €500 but shipping is charged: the row p = T, q = F. If p is false, nothing was promised.

Your Turn: A Broken Promise?

Question

"If the order exceeds €500, shipping is free." Which of these cases **breaks** the promise?

(a) Order €600, shipping charged
(b) Order €300, shipping charged
(c) Order €300, shipping free
(d) Order €600, shipping free

. . .

Only **(a)**: premise true, conclusion false, the one row where $p \Rightarrow q$ is F.

. . .

Common Mistake

Cases (b) and (c) do not break the promise: for an order of €300, nothing was promised at all. An implication with a false premise is **true**, whatever the conclusion.

Equivalence

$p \Leftrightarrow q$: "p **if and only if** q": both have the same truth value, i.e. $p \Rightarrow q$ **and** $q \Rightarrow p$.

p	q	$p \Leftrightarrow q$
T	T	**T**
T	F	F
F	T	F
F	F	**T**

. . .

Common Mistake

$\Rightarrow$ and $\Leftrightarrow$ are not interchangeable: $x = 2 \Rightarrow x^2 = 4$ holds, but $x^2 = 4 \Rightarrow x = 2$ does **not** (it could be -2).

Necessary and Sufficient Conditions

Given $p \Rightarrow q$:

- p is a sufficient condition for q: p alone guarantees q
- q is a necessary condition for p: without q, p is impossible

. . .

"If someone is a millionaire (p), then they are rich (q)."

- Being a millionaire is **sufficient** for being rich: no further criteria needed
- Being rich is **necessary** for being a millionaire, but you can be rich *without* being a millionaire

Your Turn: Necessary or Sufficient?

Question

"A container clears the port **only if** its customs declaration is valid." Is a valid declaration a **necessary** or a **sufficient** condition for clearance?

. . .

Necessary. The sentence says: clearance $\Rightarrow$ valid declaration. Without a valid declaration, clearance is impossible.

. . .

Common Mistake

It is **not** sufficient: a valid declaration alone does not guarantee clearance, an inspection or unpaid duties can still stop the container. "Only if" points to a necessary condition, "if" to a sufficient one.

Quantifiers

Two symbols turn open sentences into statements:

- $\forall$, for all: $\quad \forall x \in \mathbb{R} : x^2 \geq 0$
- $\exists$, there exists: $\quad \exists n \in \mathbb{N} : n > 10^6$

. . .

In business language:

- "**Every** route is profitable" $\rightarrow \forall$ statement
- "**Some** supplier delivers late" $\rightarrow \exists$ statement

Negating Quantified Statements

Negation swaps the quantifier and negates the inside:

$$\neg\,(\forall x : P(x)) \Leftrightarrow \exists x : \neg P(x)$$

$$\neg\,(\exists x : P(x)) \Leftrightarrow \forall x : \neg P(x)$$

. . .

- To disprove "**all** routes are profitable", one loss-making route suffices
- To disprove "**some** supplier delivers late", you must check every supplier

Your Turn: Negate a Statement

Question

Negate: "**All** shipments arrive on time."

. . .

"**At least one** shipment does *not* arrive on time."

. . .

Common Mistake

The negation is **not** "no shipment arrives on time": that is far too strong. One single late shipment already makes the original statement false.

Common Pitfall: The Converse Error

We know: "If a shipment is delayed (p), the customer complains (q)."

Today, a customer complained. **Was the shipment delayed?**

. . .

- Tempting, but we cannot conclude that
- Complaints can have other causes: damaged goods, wrong invoice, …
- From $p \Rightarrow q$ and q, **nothing** follows about p

. . .

Common Mistake

Concluding p from $p \Rightarrow q$ and q is the converse error ("affirming the consequent"), one of the most frequent reasoning errors in business and in exams.

What You *May* Conclude

From $p \Rightarrow q$ ("delayed $\Rightarrow$ complaint"):

Derived statement	Form	Valid?
No complaint $\Rightarrow$ not delayed	contrapositive $\neg q \Rightarrow \neg p$	✓
Complaint $\Rightarrow$ delayed	converse $q \Rightarrow p$	×
Not delayed $\Rightarrow$ no complaint	inverse $\neg p \Rightarrow \neg q$	×

. . .

Tip

The contrapositive is always equivalent to the original implication: flip **and** negate. Flipping alone (converse) or negating alone (inverse) breaks the logic.

Closing

Key Takeaways

- Mathematical notation is a precise, universal language
- Number sets form a hierarchy: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$
- $\sum$ and $\prod$ are compact loops for adding and multiplying
- Indices like x_{ij} carry meaning: always check their definition
- Implication is a one-way street: beware the converse error
- Negating $\forall$ gives $\exists$, and vice versa

That's it for today.

Any Questions?

Until the Next Session

- Work through the Tasks: problems with worked solutions
- Check yourself with the Self-Test quiz
- Keep the Cheatsheet next to you while practising
- Note down anything unclear: we start next session with **your questions**

. . .

Tip

The tasks are where the actual learning happens: the lecture only opens the door.

Preview: Session 02: Sets

- What sets are and how to describe them
- Operations: union, intersection, difference, complement
- Venn diagrams as a thinking tool

- Why sets are the foundation for **functions** in Session 03

. . .

See you there, and bring your questions!

Literature & Further Reading

- These sessions cover the essentials: textbooks offer more depth and practice
- Start with K. Sydsæter, P. J. Hammond, and A. Strøm [1] or I. Jacques [2]; full recommendations on the tutorial's literature page

. . .

> **ℹ Note**
>
> Mathematics also has formal proof techniques: direct proof, proof by contradiction, and induction. They are beyond the scope of this tutorial, but the textbooks above cover them if you are curious.

Bibliography

[1] K. Sydsæter, P. J. Hammond, and A. Strøm, *Essential Mathematics for Economic Analysis*, 4th ed. Harlow: Pearson, 2012.

[2] I. Jacques, *Mathematics for Economics and Business*, 8th ed. in Always Learning. Harlow: Pearson, 2015, p. 660.