Cheatsheet 01 - Notation & Logic

Mathematics for Master Students

Number Systems

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Symbol	Name	Definition	Examples
$\mathbb{N}$	Natural numbers	$\{1, 2, 3, \ldots\}$	$1, 7, 42$
$\mathbb{Z}$	Integers	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$	$-5, 0, 3$
$\mathbb{Q}$	Rational numbers	$\left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$	$\frac{1}{3}, 0.75, -4$
$\mathbb{R}$	Real numbers	All points on the number line	$\sqrt{2}, \pi$, all of the above

⚠ Common Mistakes

- In this tutorial $0 \notin \mathbb{N}$: write $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$ if zero is included.
- $\sqrt{2}$ and π are **irrational** (real, but no fraction equals them): $\pi \approx \frac{22}{7}$, but $\pi \neq \frac{22}{7}$.

Intervals & Inequalities

Notation	Inequality	Description
$[a, b]$	$a \leq x \leq b$	Closed: both endpoints included
(a, b)	$a < x < b$	Open: both endpoints excluded
$[a, b)$	$a \leq x < b$	Half-open: a included, b excluded
$(a, b]$	$a < x \leq b$	Half-open: a excluded, b included
$[a, \infty), (-\infty, a]$	$x \geq a, x \leq a$	Unbounded: the ∞ side is always open

Absolute value: the distance of x from zero ($|a - b|$ = distance between a and b):

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \qquad |x - a| \leq d \quad \Leftrightarrow \quad x \in [a - d, a + d]$$

Remembering Brackets

Square bracket [= endpoint **included** (it "grabs" it); **round parenthesis** (= **excluded**. ∞ always gets a parenthesis. It is not a number you could include.

Sum & Product Notation

$$\sum_{i=k}^{m} x_i = x_k + x_{k+1} + \ldots + x_m \qquad \prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot \ldots \cdot x_n$$

Read $\sum$ as a loop: for each **index** i from the **lower limit** k to the **upper limit** m, add up the **summand** x_i. The index name is arbitrary; $\prod$ works the same, but multiplies.

$$\sum_{i=1}^{n} c\, x_i = c \sum_{i=1}^{n} x_i \qquad \sum_{i=1}^{n} (x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i \qquad \sum_{i=1}^{n} c = n \cdot c$$

Double indices: x_{ij} = flow from origin i (first index, row) to destination j (second index, column).

- Leaving origin i: $\sum_{j=1}^{m} x_{ij}$
- Reaching destination j: $\sum_{i=1}^{n} x_{ij}$
- Total cost over all routes: $\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}\, x_{ij}$

> **Off-by-One**
>
> $\sum_{i=k}^{m} x_i$ has $m-k+1$ terms. Both endpoints count. $\sum_{i=3}^{10} x_i$ has 8 terms, not 7.

Greek Letters

Letter	Name	Typical use	Letter	Name	Typical use
α	alpha	significance level	μ	mu	mean
β	beta	regression	π	pi	$3.14159...$, profit
δ, Δ	delta	change, difference	ρ	rho	correlation
ε	epsilon	error term	σ, Σ	sigma	std. dev., sum
λ	lambda	arrival rate	θ	theta	parameter

Logic

A **statement** is either true or false, never both. Double negation cancels: $\neg(\neg p) = p$. The "or" is **inclusive**; $p \Rightarrow q$ is false only for the broken promise: p true, q false.

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

De Morgan's laws: negation flips "and" into "or", and vice versa:

$$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q \qquad \neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$$

Derived from $p \Rightarrow q$: p is **sufficient** for q; q is **necessary** for p.

Derived statement	Form	Equivalent to $p \Rightarrow q$?
Contrapositive	$\neg q \Rightarrow \neg p$	✓
Converse	$q \Rightarrow p$	✗
Inverse	$\neg p \Rightarrow \neg q$	✗

> **Common Mistakes**
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> - **Converse error:** from $p \Rightarrow q$ and q, **nothing** follows about p.
> - The negation of "the warehouse is full" is "the warehouse is **not full**", not "empty".
> - $\Rightarrow$ and $\Leftrightarrow$ are not interchangeable: $x = 2 \Rightarrow x^2 = 4$ holds, but $x^2 = 4 \Rightarrow x = 2$ fails ($x = -2$), so $x^2 = 4 \Leftrightarrow x = 2$ is false.

Quantifiers

Symbol	Meaning	Example
$\forall$	for all	$\forall x \in \mathbb{R} : x^2 \geq 0$
$\exists$	there exists	$\exists n \in \mathbb{N} : n > 10^6$

Negation swaps the quantifier and negates the predicate. To disprove $\forall$: **one counterexample**; to prove $\exists$: **one witness**.

$$\neg\left(\forall x : P(x)\right) \Leftrightarrow \exists x : \neg P(x) \qquad \neg\left(\exists x : P(x)\right) \Leftrightarrow \forall x : \neg P(x)$$

> **Common Mistake**
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> "All shipments arrive on time" negates to "**at least one** shipment does *not* arrive on time", **not** to "no shipment arrives on time". One late shipment already refutes the claim.